Emmanuel College

MA 242 – Multivariable Calculus

Exercises

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The following is a collection of exercises. Blue links lead to more detail; red links lead back.

data

topics

points/vectors

#1 $(0,0) = \mathbf{0} \in \mathbb{R}^2$

#2 $(1,0) = \mathbf{i} \in \mathbb{R}^2$

#3 $(0,1) = \mathbf{j} \in \mathbb{R}^2$

#4 $(1,2) = \mathbf{i} + 2\mathbf{j} \in \mathbb{R}^2$

#5 $(-1,-2) = -\mathbf{i} - 2\mathbf{j} \in \mathbb{R}^2$

#6 $(3,0) = 3\mathbf{i} \in \mathbb{R}^2$

#7 $(2,-2) = 2\mathbf{i} - 2\mathbf{j} \in \mathbb{R}^2$

#8 $(8,7) = 8\mathbf{i} + 7\mathbf{j} \in \mathbb{R}^2$

#9 $(-4,-5) = -4\mathbf{i} - 5\mathbf{j} \in \mathbb{R}^2$

#10 $(0,0,0) = \mathbf{0} \in \mathbb{R}^3$

#11 $(1,0,0) = \mathbf{i} \in \mathbb{R}^3$

#12 $(0,1,0) = \mathbf{j} \in \mathbb{R}^3$

#13 $(0,0,1) = \mathbf{k} \in \mathbb{R}^3$

#14 $(1,2,3) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \in \mathbb{R}^3$

#15 $(-1,-2,2) = -\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \in \mathbb{R}^3$

#16 $(3,0,-4) = 3\mathbf{i} - 4\mathbf{k} \in \mathbb{R}^3$

#17 $(2,-2,-2) = 2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k} \in \mathbb{R}^3$

#18 $(8,7,1) = 8\mathbf{i} + 7\mathbf{j} + \mathbf{k} \in \mathbb{R}^3$

#19 $(-4,-5,-6) = -4\mathbf{i} - 5\mathbf{j} - 6\mathbf{k} \in \mathbb{R}^3$

#20 $(10, 0, 0) = 10\mathbf{i} \in \mathbb{R}^3$

#21 $(0, -1, -10) = -\mathbf{j} - 10\mathbf{k} \in \mathbb{R}^3$

#22 $(5, 5, 5) = 5\mathbf{i} + 5\mathbf{j} + 5\mathbf{k} \in \mathbb{R}^3$

#23 $(5, -5, 5) = 5\mathbf{i} - 5\mathbf{j} + 5\mathbf{k} \in \mathbb{R}^3$

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curves

#1 the line $x = y$ in $\mathbb{R}^2$

#2 the line $x + y = 1$ in $\mathbb{R}^2$

#3 the line $x + 2y = 3$ in $\mathbb{R}^2$

#4 the line $x = y = z$ in $\mathbb{R}^3$

#5 the line $x = 2y = 3z$ in $\mathbb{R}^3$

#6 the line $x + 2 = 2y + 3 = 3z + 4$ in $\mathbb{R}^3$

#7 the segment joining $(0, 0)$ and $(1, 1)$ in $\mathbb{R}^2$

#8 the segment joining $(1, 2)$ and $(-3, 4)$ in $\mathbb{R}^2$

#9 the segment joining $(0, 0, 0)$ and $(1, 1, 1)$ in $\mathbb{R}^3$

#10 the segment joining $(1, -2, 3)$ and $(-4, 5, 6)$ in $\mathbb{R}^3$

#11 the parabola $x^2 = y$ in $\mathbb{R}^2$

#12 the parabola $x + y^2 = 3$ in $\mathbb{R}^2$

#13 the circle $x^2 + y^2 = 1$ in $\mathbb{R}^2$

#14 the ellipse $x^2 + 4y^2 = 9$ in $\mathbb{R}^2$

#15 the circle $(y - 1)^2 + (z + 2)^2 = 1, x = 1$ in $\mathbb{R}^3$

#16 the ellipse $(z + 1)^2 + 4(x - 2)^2 = 9, y = -1$ in $\mathbb{R}^3$

#17 the circle $x^2 + y^2 = 1, z = 1$ in $\mathbb{R}^3$

#18 the ellipse $y^2 + 4z^2 = 9, x = -1$ in $\mathbb{R}^3$

#19 the circle $(z - 1)^2 + (x + 2)^2 = 1, y = 1$ in $\mathbb{R}^3$

#20 the ellipse $(x+1)^2 + 4(y-2)^2 = 9, z = -1$ in $\mathbb{R}^3$

#21 the segment $(-1+t, -2-t, 1-3t), t \in [-2, 5]$ in $\mathbb{R}^3$

#22 the helix $(\cos t, \sin t, t), t \in [0, 4\pi]$ in $\mathbb{R}^3$

#23 the elliptical helix $(t, 2\cos(2\pi t), 3\sin(2\pi t)), t \in [-2, 2]$ in $\mathbb{R}^3$

#24 $(t, t^2, t^3), t \in [0, 1]$ in $\mathbb{R}^3$

#25 $(t, \sin t, e^{\frac{t}{\pi}}), t \in [0, 2\pi]$ in $\mathbb{R}^3$

#26 $(2t^2, e^t, \sin \pi t), t \in [0, 2\pi]$ in $\mathbb{R}^3$

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surfaces

#1 the plane $x + y + z = 1$ in $\mathbb{R}^3$

#2 the plane $x + 2y + 3z = 4$ in $\mathbb{R}^3$

#3 the plane $3x - 2y - z = 0$ in $\mathbb{R}^3$

#4 the plane $-x - 2y + 3z = -4$ in $\mathbb{R}^3$

#5 the square region $[0, 1] \times [0, 1]$ in $\mathbb{R}^2$

#6 the triangular region $x + y \leq 1, x \geq 0, y \geq 0$ in $\mathbb{R}^2$

#7 the disk $x^2 + y^2 \leq 1$ in $\mathbb{R}^2$

#8 the elliptical disk $x^2 + 4y^2 \leq 9$ in $\mathbb{R}^2$

#9 the region described by $0 \leq y \leq 4 - x^2$ in $\mathbb{R}^2$

#10 the region described by $x^2 \leq y \leq 4 - x^2$ in $\mathbb{R}^2$

#11 the region described by $y^2 \leq x \leq y$ in $\mathbb{R}^2$

#12 the region described by $y^3 \leq x \leq y^2$ in $\mathbb{R}^2$

#13 the region described by $1 - x^2 \leq y \leq e^x$ in $\mathbb{R}^2$

#14 the disk $(y - 1)^2 + (z + 2)^2 \leq 1, x = 1$ in $\mathbb{R}^3$

#15 the square surface $[0, 1] \times [0, 1] \times \{0\}$ in $\mathbb{R}^3$

#16 the triangular surface $x + y + z = 1, x \geq 0, y \geq 0, z \geq 0$ in $\mathbb{R}^3$

#17 the elliptical disk $(z + 1)^2 + 4(x - 2)^2 \leq 9, y = -1$ in $\mathbb{R}^3$

#18 the disk $x^2 + y^2 \leq 1, z = 1$ in $\mathbb{R}^3$

#19 the elliptical disk $y^2 + 4z^2 \leq 9, x = -1$ in $\mathbb{R}^3$

#20 the disk $(z-1)^2 + (x+2)^2 \leq 1, y = 1$ in $\mathbb{R}^3$

#21 the elliptical disk $(x+1)^2 + 4(y-2)^2 \leq 9, z = -1$ in $\mathbb{R}^3$

#22 the saddle $xy = z, (x, y) \in [-1, 1]^2$ in $\mathbb{R}^3$

#23 the saddle $xy = z, x^2 + y^2 \leq 1$ in $\mathbb{R}^3$

#24 the hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$ in $\mathbb{R}^3$

#25 the paraboloid $x^2 + y^2 = z, z \leq 1$ in $\mathbb{R}^3$

#26 the cone $x^2 + y^2 = z^2, z \leq 1$ in $\mathbb{R}^3$

#27 the cylinder $x^2 + y^2 = 1, -1 \leq z \leq 2$ in $\mathbb{R}^3$

#28 the elliptical cylinder $4y^2 + z^2 = 4, 0 \leq x \leq 1$ in $\mathbb{R}^3$

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solids

#1 the solid cube $[0,1] \times [0,1] \times [0,1]$

#2 the solid cube $[-1,1] \times [-1,1] \times [-1,1]$

#3 The tetrahedron with vertices $(3,0,0), (0,2,0), (0,0,1), (0,0,0)$

#4 the ball $x^2 + y^2 + z^2 \leq 1$

#5 the half-ball $x^2 + y^2 + z^2 \leq 1, z \geq 0$

#6 the quarter-ball $x^2 + y^2 + z^2 \leq 1, y \geq 0, z \geq 0$

#7 the eighth-ball $x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0, z \geq 0$

#8 the solid paraboloid $x^2 + y^2 \leq z \leq 1$

#9 the solid cone $x^2 + y^2 \leq z^2 \leq 1$

#10 the solid cylinder $x^2 + y^2 \leq 1, -1 \leq z \leq 2$ in $\mathbb{R}^3$

#11 the solid elliptical cylinder $4y^2 + z^2 \leq 4, 0 \leq x \leq 1$ in $\mathbb{R}^3$

#12 the region in $\mathbb{R}^3$ bounded by cylinder $z = y^2$ and planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$

#13 the region in $\mathbb{R}^3$ bounded by planes $x = 0$, $y = 0$, $z = 0$, $x + z = 1$, $y + 2z = 2$

#14 the region in $\mathbb{R}^3$ bounded by planes $x = 0$, $y = 0$, $z = 0$, $y + z = 1$, $x = 4 - y^2$

#15 the region in $\mathbb{R}^3$ bounded by planes $z = 0$, $y = -z$ and cylinder $x^2 + y^2 = 1$

#16 the region in $\mathbb{R}^3$ bounded by planes $x = 0$, $y = 0$, $z = 0$, plane $x + y = 1$, and surface $z = \cos(\pi y/2)$

#17 the region in $\mathbb{R}^3$ bounded by the cylinders $x^2+y^2 \leq 1$ and $y^2+z^2 \leq 1$

#18 the region in $\mathbb{R}^3$ bounded by the planes $x = 0$, $y = 0$, $z = 0$ and the surface $x + y + z^2 = 4$

#19 the region in $\mathbb{R}^3$ bounded by the planes $x = 0$, $y = 0$, $z = 0$, $y+z = 4$ and the elliptical cylinder $4x^2 + y^2 = 16$

#20 the region in $\mathbb{R}^3$ bounded by the planes $z = 0$, $y + z = 3$ and the cylinder $x^2 + y^2 = 4$

#21 the region in $\mathbb{R}^3$ bounded by the planes $x = 0$, $z = 0$ and the surfaces $x^2 + y = 1$ and $x^2 + y^2 = z$

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multivariable functions

#1 $f(x,y) = x + y$

#2 $f(x,y) = x^2 + y^2$

#3 $f(x,y) = xy$

#4 $f(x,y) = x + 2y$

#5 $f(x,y) = x^2 + 2y^2$

#6 $f(x,y,z) = x + 2y + 3z$

#7 $f(x,y,z) = x + y + z$

#8 $f(x,y,z) = x^2 + 2y^2 + 3z^2$

#9 $f(x,y,z) = x^2 + y^2 + z^2$

#10 $f(x,y,z) = xyz$

#11 $\mathbf{F}(x,y) = (1,1) = \mathbf{i} + \mathbf{j}$

#12 $\mathbf{F}(x,y) = (x,y) = x\mathbf{i} + y\mathbf{j}$

#13 $\mathbf{F}(x,y) = (y,x) = y\mathbf{i} + x\mathbf{j}$

#14 $\mathbf{F}(x,y) = (\cos x, e^y) = \cos x\mathbf{i} + e^y\mathbf{j}$

#15 $\mathbf{F}(x,y) = (\frac{y}{x}x^y, (\ln x)x^y) = \frac{y}{x}x^y\mathbf{i} + (\ln x)x^y\mathbf{j}$

#16 $\mathbf{F}(x,y) = (x\cos y, x\sin y) = x\cos y\mathbf{i} + x\sin y\mathbf{j}$

#17 $\mathbf{F}(x,y,z) = (yz, xz, xy) = yz\mathbf{i} + xz\mathbf{k} + xy\mathbf{k}$

#18 $\mathbf{F}(x,y,z) = (e^x\cos y, -e^x\sin y, z) = (e^x\cos y)\mathbf{i} - (e^x\sin y)\mathbf{j} + z\mathbf{k}$

#19 $\mathbf{F}(x,y,z) = (x, 2y, 3z) = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$

#20 $\mathbf{F}(x,y,z) = (3y, 2x, z) = 3y\mathbf{i} + 2x\mathbf{j} + z\mathbf{k}$

#21 $\mathbf{F}(x,y,z) = (-3x, 5y, -7z) = -3x\mathbf{i} + 5y\mathbf{j} - 7z\mathbf{k}$

#22 $\mathbf{F}(x,y,z) = (-5y, 7x, 11z) = -5y\mathbf{i} + 7x\mathbf{j} + 11z\mathbf{k}$

#23 $\mathbf{F}(x,y,z) = (x^2+y, y^2+z, z^2) = (x^2+y)\mathbf{i} + (y^2+z)\mathbf{j} + z^2\mathbf{k}$

#24 $\mathbf{F}(x,y,z) = (x\cos z, y\sin z, z^2) = x\cos z\mathbf{i} + y\sin z\mathbf{j} + z^2\mathbf{k}$

#25 $\mathbf{F}(x,y,z) = (y-x, z-y, y-x) = (y-x)\mathbf{i} + (z-y)\mathbf{j} + (y-x)\mathbf{k}$

#26 $\mathbf{F}(x,y,z) = (z+y, z, y+x) = (z+y)\mathbf{i} + z\mathbf{j} + (y+x)\mathbf{k}$

#27 $\mathbf{F}(x,y,z) = (z+y, x+z, y+x) = (z+y)\mathbf{i} + (x+z)\mathbf{j} + (z+y)\mathbf{k}$

#28 $\mathbf{F}(x,y,z) = (x, -2y, z+3) = x\mathbf{i} - 2y\mathbf{j} + (z+3)\mathbf{k}$

#29 $\mathbf{F}(x,y,z) = (12x, -y^2-4yz, -6zx) = 12x\mathbf{i} - (y^2+4yz)\mathbf{j} - 6zx\mathbf{k}$

#30 $\mathbf{F}(x,y,z) = (x+1, y-1, z+2) = (x+1)\mathbf{i} + (y-1)\mathbf{j} + (z+2)\mathbf{k}$

#31 $\mathbf{F}(x,y,z) = (x+y, 2y, z-x) = (x+y)\mathbf{i} + 2y\mathbf{j} + (z-x)\mathbf{k}$

#32 $\mathbf{F}(x,y,z) = (y, 2y-z, z+3x) = y\mathbf{i} + (2y-z)\mathbf{j} + (z+3x)\mathbf{k}$

#33 $\mathbf{F}(x,y,z) = (x\cos y, x\sin y, z) = x\cos y\mathbf{i} + x\sin y\mathbf{j} + z\mathbf{k}$

#34 $\mathbf{F}(x,y,z) = (x\sin y\cos z, x\sin y\sin z, x\cos y)$
$= x\sin y\cos z\mathbf{i} + x\sin y\sin z\mathbf{j} + x\cos y\mathbf{k}$

#35 $\mathbf{F}(x,y,z) = (y\sin z, x\sin z, xy\cos z) = (y\sin z)\mathbf{i}+(x\sin z)\mathbf{j}+(xy\cos z)\mathbf{k}$

vectors

#1 For each vector $\mathbf{v}$ in the **data** section, calculate

(a) the length of $\mathbf{v}$.

(b) the unit vector in the same direction as $\mathbf{v}$.

#2 For each pair of $\mathbb{R}^2$-vectors $\mathbf{v}, \mathbf{w}$ in the **data** section, calculate

(a) the angle between $\mathbf{v}$ and $\mathbf{w}$.

(b) the scalar projection of $\mathbf{v}$ onto $\mathbf{w}$.

(c) the vector projection of $\mathbf{v}$ onto $\mathbf{w}$.

#3 For each pair of $\mathbb{R}^3$-vectors $\mathbf{v}, \mathbf{w}$ in the **data** section, calculate

(a) the angle between $\mathbf{v}$ and $\mathbf{w}$.

(b) the scalar projection of $\mathbf{v}$ onto $\mathbf{w}$.

(c) the vector projection of $\mathbf{v}$ onto $\mathbf{w}$.

(d) the cross product $\mathbf{v} \times \mathbf{w}$ of $\mathbf{v}$ and $\mathbf{w}$.

#4 For each triple of $\mathbb{R}^3$-vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in the **data** section, calculate the triple scalar product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.

points, lines, planes, spheres

#1 For each pair of $\mathbb{R}^2$-points $\mathbf{a}, \mathbf{b}$ in the **data** section, find
(a) the vector from $\mathbf{a}$ to $\mathbf{b}$.
(b) the distance from $\mathbf{a}$ to $\mathbf{b}$.

#2 For each pair of $\mathbb{R}^3$-points $\mathbf{a}, \mathbf{b}$ in the **data** section, find
(a) the vector from $\mathbf{a}$ to $\mathbf{b}$.
(b) the distance from $\mathbf{a}$ to $\mathbf{b}$.

#3 For each pair of $\mathbb{R}^2$-points $\mathbf{a}, \mathbf{b}$ in the **data** section, find the line containing $\mathbf{a}$ and $\mathbf{b}$, written
(a) in terms of the parameter t.
(b) as an equation involving x, y.

#4 For each pair of $\mathbb{R}^3$-points $\mathbf{a}, \mathbf{b}$ in the **data** section, find the line containing $\mathbf{a}$ and $\mathbf{b}$, written
(a) in terms of the parameter t.
(b) as an equation involving x, y, z.

#5 For each triple of $\mathbb{R}^3$-points $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in the **data** section, find the plane containing $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, written
(a) in terms of the parameters u, v.
(b) as an equation involving x, y, z.

#6 For each point and plane in the **data** section, find the (least) distance from the point to the plane.

#7 For each $\mathbb{R}^2$-point $\mathbf{p}$ in the **data** section, find the equation of the circle with center $\mathbf{p}$ and
(a) having radius 1.
(b) having radius 2.
(c) containing the point $(2, 3)$.
(d) tangent to the line $x + y = 7$.

#8 For each $\mathbb{R}^3$-point $\mathbf{p}$ in the **data** section, find the equation of the sphere with center $\mathbf{p}$ and

(a) having radius 1.
(b) having radius 2.
(c) containing the point $(2, 3, 4)$.
(d) tangent to the plane $x + y + z = 7$.

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point/vector-valued functions

#1 For each scalar-valued $\mathbb{R}^2$-function f and each vector-valued $\mathbb{R}^2$-function $\mathbf{F}$ in the **data** section, calculate

$$f(\mathbf{F}(x,y)).$$

#2 For each scalar-valued $\mathbb{R}^3$-function f and each vector-valued $\mathbb{R}^3$-function $\mathbf{F}$ in the **data** section, calculate

$$f(\mathbf{F}(x,y,z)).$$

motion

#1 For each curve in the **data** section,

(a) express the curve as the trajectory of a moving particle with position $\mathbf{r}(t)$ at time t.

(b) calculate the velocity of the particle at time t.

(c) calculate the speed of the particle at time t.

(d) calculate the distance traveled by the particle up to time t.

(e) calculate the acceleration of the particle at time t.

(f) suppose that $\mathbf{r}(t) = \mathbf{w}'(t)$ for some curve $\mathbf{w}(t)$ with $\mathbf{w}(0) = (3, 2, 1)$. Find the parametric equation $\mathbf{w}(t)$ of the curve.

(g) suppose that $\mathbf{r}(t) = \mathbf{w}''(t)$ for some curve $\mathbf{w}(t)$ with $\mathbf{w}(0) = (3, 2, 1)$ and $\mathbf{w}'(0) = (-1, -2, -3)$. Find the parametric equation $\mathbf{w}(t)$ of the curve.

the Frenet Frame

#1 For each curve in the **data** section, calculate the Frenet Frame at each point of the curve.

#2 For each curve in the **data** section, calculate the torsion at each point of the curve.

#3 For each curve in the **data** section, calculate the curvature at each point of the curve.

#4 Considering each curve in the **data** section as the trajectory of a moving particle, calculate the tangential and normal components of the particle's motion at each point of its trajectory.

note: Your answer to this question will depend on how you parametrize the curve.

functions of several variables

#1 Find

$$\lim_{(x,y)\to(0,0)} \frac{x}{x+y}$$

if it exists.

#2 Find

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x+y}$$

if it exists.

#3 Find

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

if it exists.

#4 Find

$$\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$$

if it exists.

#5 Find

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{\sqrt{x^2+y^2}}$$

if it exists.

#6 Find

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$$

if it exists.

#7 Find

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y}{\sqrt{x^2+y^2}}$$

if it exists.

#8 For each $\mathbb{R}^2$-function $f(\mathbf{x})$ and each $\mathbb{R}^2$-curve in the **data** section, parametrize the curve as $\mathbf{r}(t)$ and find the derivative of the function $f(\mathbf{r}(t))$.

#9 For each $\mathbb{R}^3$-function $f(\mathbf{x})$ and each $\mathbb{R}^3$-curve in the **data** section, parametrize the curve as $\mathbf{r}(t)$ and find the derivative of the function $f(\mathbf{r}(t))$.

#10 Consider the function

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases} .$$

(a) Calculate the mixed partial derivative

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

at $(0,0)$.

(b) Calculate the mixed partial derivative

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

at $(0,0)$.

(c) Why do these results not violate Clairaut's Theorem?

#11 For each function $f(\mathbf{x})$ on $\mathbb{R}^2$ and each vector $\mathbf{v} \in \mathbb{R}^2$ in the **data** section, calculate the directional derivative of $f(\mathbf{x})$ in the direction of $\mathbf{v}$.

#12 For each function $f(\mathbf{x})$ on $\mathbb{R}^3$ and each vector $\mathbf{v} \in \mathbb{R}^3$ in the **data** section, calculate the directional derivative of $f(\mathbf{x})$ in the direction of $\mathbf{v}$.

#13 For each function $f(\mathbf{x})$ on $\mathbb{R}^2$ and each point $\mathbf{p} \in \mathbb{R}^2$ in the **data** section, find the linear approximation of $f(\mathbf{x})$ about $\mathbf{p}$.

#14 For each function $f(\mathbf{x})$ on $\mathbb{R}^3$ and each point $\mathbf{p} \in \mathbb{R}^3$ in the **data** section, find the linear approximation of $f(\mathbf{x})$ about $\mathbf{p}$.

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extrema

#1 For each function $f(\mathbf{x})$ in the **data** section, calculate its global extrema on its domain, if they exist.

#2 For each $\mathbb{R}^2$-function $f(x, y)$ and and $\mathbb{R}^2$-region K in the **data** section, calculate the global extrema of

$$f|_K(x, y)$$

(the function restricted to the region).

#3 For each $\mathbb{R}^3$-function $f(x, y, z)$ and and $\mathbb{R}^3$-region W in the **data** section, calculate the global extrema of

$$f|_W(x, y, z)$$

(the function restricted to the region).

#4 For each $\mathbb{R}^2$=function $f(\mathbf{x})$ in the **data** section, calculate its local extrema and saddle points, if they exist.

#5 For each bounded surface $\mathbf{R}$ in the **data** section, use the method of Lagrange multipliers to find the minimum distance between
(a) $\mathbf{R}$ and the point $10\mathbf{i}$.
(b) $\mathbf{R}$ and the point $10\mathbf{j}$.
(c) $\mathbf{R}$ and the point $10\mathbf{k}$.

#6 Use the method of Lagrange Multipliers to find the maximum area of a rectangular piece of land surrounded by a total 1000m of fence.

#7 Use the method of Lagrange Multipliers to find the maximum area of a rectangular piece of land bordering on a straight river on one side and surrounded by a total 1000m of fence on the other three sides.

#8 Use the method of Lagrange Multipliers to find the maximum volume of a closed rectangular box made of 1000m^2 of sheet metal.

#9 Use the method of Lagrange Multipliers to find the maximum volume of an open rectangular box made of 1000m^2 of sheet metal.

#10 Use the method of Lagrange Multipliers to find the maximum volume of a closed (circular) cylindrical tank made of 1000m^2 of sheet metal.

#11 Use the method of Lagrange Multipliers to find the maximum volume of an open (circular) cylindrical tank made of 1000m^2 of sheet metal.

#12 A stream, 10m wide, runs east-west and has negligible current. Running along the north edge of the stream is a road, 10m wide. A frog is sitting on the north side of the road, and wants get to the south side of the stream, and 20m downstream, as quickly as possible. Use the method of Lagrange Multipliers to calculate the shortest possible time it will take the frog, who can hop 0.4m/s on land and swim 0.6m/s in the water. (Assume that the road is free of traffic and that the stream is free of alligators.)

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double integrals

#1 For each scalar-valued $\mathbb{R}^2$-function f and $\mathbb{R}^2$-region K in the **data** section,
(a) set up

$$\iint\limits_K f \, dA$$

as an iterated integral.
(b) evaluate the integral.

triple integrals

#1 For each scalar-valued $\mathbb{R}^3$-function f and $\mathbb{R}^3$-region W in the **data** section,

(a) set up

$$\iiint\limits_W f \, dV$$

as an iterated integral.

(b) evaluate the integral.

polar coordinates

#1 Express each of the scalar-valued $\mathbb{R}^2$-functions in the **data** section, in terms of polar coordinates.

#2 Describe each of the $\mathbb{R}^2$-regions in the **data** section, in terms of polar coordinates.

#3 For each scalar-valued $\mathbb{R}^2$-function $f(x, y)$ and each $\mathbb{R}^2$-region K in the **data** section,

(a) write

$$\iint_K f dA$$

as an iterated integral in polar coordinates.

(b) evaluate the integral.

cylindrical and spherical coordinates

#4 For each scalar-valued $\mathbb{R}^3$-function f and $\mathbb{R}^3$-region W in the **data** section,
(a) set up

$$\iiint_W f \, dV$$

as an iterated integral in cylindrical coordinates.
(b) evaluate the integral.

#5 For each scalar-valued $\mathbb{R}^3$-function f and $\mathbb{R}^3$-region W in the **data** section,
(a) set up

$$\iiint_W f \, dV$$

as an iterated integral in spherical coordinates.
(b) evaluate the integral.

substitution

#1 For each change of variables **F** in the **data** section, calculate the Jacobian determinant

$$\left|\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right|.$$

parametric curves and surfaces

#1 For each curve $\mathbf{r}$ in the **data** section not already written in parametric form, express $\mathbf{r}$ as a parametric curve $\mathbf{r}(t)$. Be sure to state the domain of the parameter t.

#2 For each surface $\mathbf{R}$ in the **data** section not already written in parametric form, express $\mathbf{R}$ as a parametric surface $\mathbf{R}(u, v)$. Be sure to state the domain of the parameters u and v.

scalar functions and vector fields

#1 For each scalar-valued function f on $\mathbb{R}^2$ and each $\mathbb{R}^2$-curve in the **data** section, parametrize the curve as $\mathbf{r}(t)$ and calculate

$$f(\mathbf{r}(t)).$$

#2 For each vector-valued function $\mathbf{F}$ on $\mathbb{R}^2$ and each $\mathbb{R}^2$-curve in the **data** section, parametrize the curve as $\mathbf{r}(t)$ and calculate

$$\mathbf{F}(\mathbf{r}(t)).$$

#3 For each scalar-valued function f on $\mathbb{R}^3$ and each $\mathbb{R}^3$-curve in the **data** section, parametrize the curve as $\mathbf{r}(t)$ and calculate

$$f(\mathbf{r}(t)).$$

#4 For each vector-valued function $\mathbf{F}$ on $\mathbb{R}^3$ and each $\mathbb{R}^3$-curve in the **data** section, parametrize the curve as $\mathbf{r}(t)$ and calculate

$$\mathbf{F}(\mathbf{r}(t)).$$

#5 For each scalar-valued function f on $\mathbb{R}^3$ and each $\mathbb{R}^3$-surface in the **data** section, parametrize the surface as $\mathbf{R}(u, v)$ and calculate

$$f(\mathbf{R}(u, v)).$$

#6 For each vector-valued function $\mathbf{F}$ on $\mathbb{R}^3$ and each $\mathbb{R}^3$-surface in the **data** section, parametrize the surface as $\mathbf{R}(u, v)$ and calculate

$$\mathbf{F}(\mathbf{R}(u, v)).$$

new kinds of integrals

#1 For each scalar-valued function f and each bounded curve $\mathbf{r}$ in the **data** section,
(a) set up

$$\int_{\mathbf{r}} f \, ds$$

as an iterated integral.
(b) evaluate the integral.

#2 For each vector field $\mathbf{F}$ and each bounded curve $\mathbf{r}$ in the **data** section,
(a) set up

$$\int_{\mathbf{r}} \mathbf{F} \cdot \mathbf{t} \, ds$$

as an iterated integral.
(b) evaluate the integral.

#3 For each $\mathbb{R}^2$-vector field $\mathbf{F}$ and each bounded $\mathbb{R}^2$-curve $\mathbf{r}$ in the **data** section,
(a) set up

$$\int_{\mathbf{r}} \mathbf{F} \cdot \mathbf{n} \, ds$$

as an iterated integral.
(b) evaluate the integral.

#4 For each scalar-valued function f and each bounded surface $\mathbf{R}$ in the **data** section,
(a) set up

$$\iint_{\mathbf{R}} f \, dS$$

as an iterated integral.
(b) evaluate the integral.

#5 For each vector field $\mathbf{F}$ and each bounded surface $\mathbf{R}$ in the **data** section,

(a) set up

$$\iint_{\mathbf{R}} \mathbf{F} \cdot \mathbf{N} \, dS$$

as an iterated integral.

(b) evaluate the integral.

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divergence and curl

#1 For each vector field **F** in the **data** section, calculate its divergence

$$\mathrm{div}\mathbf{F}.$$

#2 For each vector field **F** in the **data** section, calculate its curl

$$\mathrm{curl}\mathbf{F}.$$

Green's, Gauss', and Stokes' Theorems

#1 For each bounded $\mathbb{R}^2$-region K and each $\mathbb{R}^2$-vector field $\mathbf{F}$ in the **data** section,
(a) specialize Green's Theorem to K and $\mathbf{F}$.
(b) verify that instance of Green's Theorem by evaluating each side of the equality directly.

#2 For each bounded $\mathbb{R}^3$-region W and each $\mathbb{R}^3$-vector field $\mathbf{F}$ in the **data** section,
(a) specialize Gauss' Theorem to W and $\mathbf{F}$.
(b) verify that instance of Gauss' Theorem by evaluating each side of the equality directly.

#3 For each bounded $\mathbb{R}^3$-surface $\mathbf{R}$ and each $\mathbb{R}^3$-vector field $\mathbf{F}$ in the **data** section,
(a) specialize Stokes' Theorem to $\mathbf{R}$ and $\mathbf{F}$.
(b) verify that instance of Stokes' Theorem by evaluating each side of the equality directly.

conservative vector fields

#1 For each vector field **F** in the **data** section,

(a) determine whether or not **F** is conservative.

(b) find a potential function for **F** if it is conservative.